Resonant and rolling droplet

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Abstract. When an oil droplet is placed on a quiescent oil bath, it eventually collapses into the bath due to gravity. The resulting coalescence may be eliminated when the bath is vertically vibrated. The droplet bounces periodically on the bath, and the air layer between the droplet and the bath is replenished at each bounce. This sustained bouncing motion is achieved when the forcing acceleration is higher than a threshold value. When the droplet has a sufficiently low viscosity, it significantly deforms: spherical harmonic Y_ℓ^m modes are excited, resulting in resonant effects on the threshold acceleration curve. Indeed, a lower acceleration is needed when ℓ modes with $m = 0$ are excited. Modes $m \neq 0$ are found to decrease the bouncing ability of the droplet. A break of degeneracy is observed for the m parameter. In particular, when the mode $\ell = 2$ and $m = 1$ is excited, the droplet rolls on the vibrated surface without touching it, leading to a new self-propulsion mode.

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Bouncing droplets on a vibrated bath were first studied by Couder *et al* [1]. With a radius $R < 0.5$ mm, they may generate local waves on a 50 cSt bath and use them to move horizontally and interact with their surroundings: they detect submarine obstacles that locally change the oil depth, they orbit together or form lattices, they are diffracted when they pass through a slit [2]–[6].

In contrast with these previous studies, we choose a bath viscosity of 1000 cSt, a droplet viscosity between 1.5 and 100 cSt and a droplet radius R between 0.76 and 0.93 mm. With a bath viscosity at least 10 times larger than the droplet viscosity, the bath deformations are inhibited (capillary waves are fully damped), while the harmonic deformations of the droplet are enhanced. As shown recently in [7], the droplet deformation ensures its bouncing ability. Various modes of deformation may be excited as depicted in figure 1. Each picture has been constructed from an experimental snapshot of the droplet (the left side of each picture) and the calculated 3D spherical harmonic (the right side). Those modes are analogous to the natural modes of deformation introduced by Rayleigh [8] and may be expressed in terms of spherical harmonics Y_ℓ^m. The simplest mode that may be used for bouncing is the mode Y_2^0 [7]. We will show that droplets can use non-axisymetric mode Y_2^1 to move horizontally, or more precisely to roll over the bath. This new mode of self-propulsion drastically contrasts with the bouncing walker mode described in [3] since bath deformations are not necessary for the propulsion.

1. Determination of the acceleration threshold for bouncing

On a bath vertically vibrated according to a sinusoidal motion $A \sin(2\pi f t)$, periodic bouncing only occurs when the reduced maximal vertical acceleration of the bath $\Gamma = 4\pi^2 A f^2/g$ is higher than a threshold value Γ_{th}, where g is the acceleration of gravity. Formally, Γ_{th} depends on the forcing frequency f [7], the droplet radius R [9], and the physical parameters of the liquids (density ρ, viscosity ν, surface tension σ). It has been shown in [7] that Γ_{th} does not depend on the air parameters. We measure the threshold Γ_{th} as a function of the forcing frequency for various droplet sizes and viscosities. For each frequency, we place a droplet on the bath in a bouncing configuration, i.e. $\Gamma > \Gamma_{\text{th}}$. The forcing acceleration is then decreased progressively. When the threshold is reached, the droplet cannot sustain periodic bouncing anymore and quickly coalesces with the bath. Since the droplet and the bath are made from different liquids, a coalescence event locally contaminates the bath. Therefore, droplets always need to be placed at different locations on the bath.

In figure 2, the threshold acceleration Γ_{th} is represented as a function of f, for droplets with viscosities $\nu = 1.5$, 10 and 100 cSt. The threshold acceleration for the 100 cSt droplet monotonically increases with the frequency. By opposition, the 1.5 cSt curve is characterized by regularly spaced local minima that correspond to a resonance of the system: a minimal energy supply is required to sustain the periodic bouncing motion. The first minimum, at $f = 50$ Hz, corresponds to $\Gamma_{\text{th}} = 0.25$, a value significantly less than the 1 g minimal threshold required by inelastic bouncing objects on a vibrated plate. Modes $m = 0$ and $\ell = 2$, 3 and 4 are observed for forcing frequencies corresponding to minima in the threshold acceleration curve. The higher the frequency, the higher order the excited mode, and the higher the corresponding threshold acceleration. We note that the asymmetric mode Y_2^1 occurs at a local maximum of the threshold curve.

The bouncing droplet may be considered as an oscillating system analogous to the damped driven harmonic oscillator: surface tension is the restoring force and viscosity the damping

Y_2^0
$f = 50\,\text{Hz}$
$\Gamma = 0.3$
1st min $\Gamma_{th}(1.5\,\text{cSt})$

Y_3^0
$f = 160\,\text{Hz}$
$\Gamma = 2$
2nd min $\Gamma_{th}(1.5\,\text{cSt})$

Y_4^0
$f = 275\,\text{Hz}$
$\Gamma = 6$
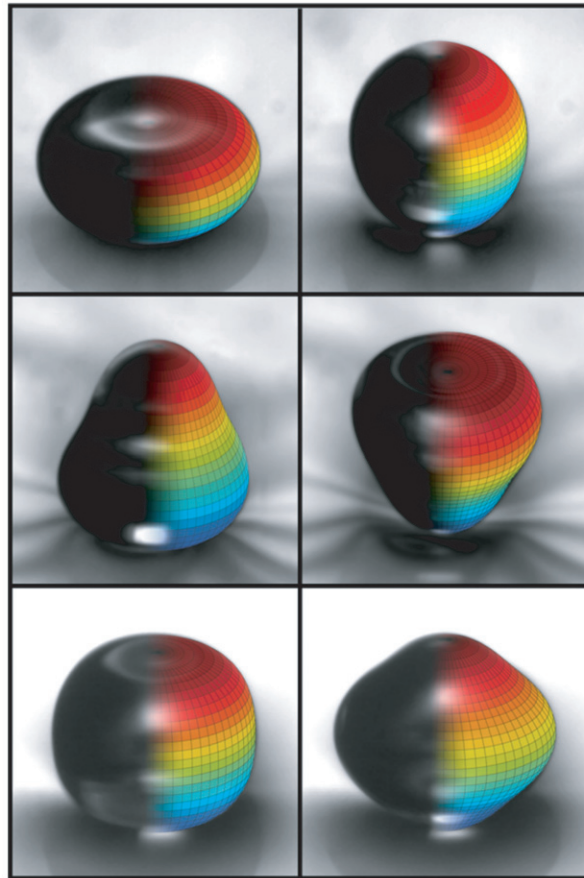3rd min $\Gamma_{th}(1.5\,\text{cSt})$

Figure 1. Various deformation modes of a bouncing droplet ($\nu = 1.5\,\text{cSt}$, $R = 0.765\,\text{mm}$) observed with a high-speed camera. The first line (respectively 2nd and 3rd) displays a mode Y_2^0 (respectively, Y_3^0 and Y_4^0) (axisymmetry). The forcing frequency f and reduced acceleration Γ corresponding to these modes are indicated in the figure. These sets of frequencies and reduced acceleration correspond to the first three minima in the curve $\Gamma_{th}(f)$ represented for $\nu = 1.5\,\text{cSt}$. The two columns represent snapshots taken at two different phases of the oscillation. The spherical harmonic solution (in colour on the right of each picture) is superposed on the experimental pictures (in grey on the left of each picture). Movie 1: available from stacks.iop.org/NJP/10/113021/mmedia. The movie compares the bouncing droplet in the modes Y_2^0, Y_3^0, and Y_4^0 with the spherical harmonic function obtained using MATLAB$^{\text{TM}}$.

process. The dimensionless ratio between the two is the Ohnesorge number $Oh = \nu\sqrt{\rho}/\sqrt{\sigma R}$, which is equal to 0.012, 0.078 and 0.775 for a droplet viscosity of 1.5, 10 and 100 cSt, respectively. When $Oh \ll 1$, the viscous damping is negligible and resonance is acute. Damping increases as Oh gets closer to 1. As seen in figure 2, the bouncing droplet reacts to an increased damping in the same way as a harmonic oscillator: (i) the resonance frequency slightly decreases, shifting the whole threshold curve to the left, (ii) the required input Γ_{th} increases and (iii) extrema tend to disappear. At 100 cSt ($Oh = 0.775$), the damping is fully active and the threshold curve increases monotonically with the frequency: no more resonance
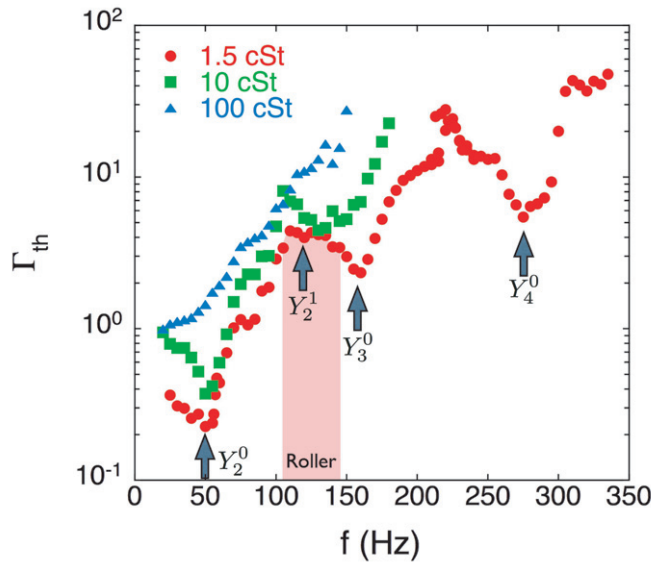
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Figure 2. Evolution of the bouncing threshold acceleration Γ_{th} with respect to the forcing frequency. The red bullets, green squares and blue triangles correspond to droplets with viscosity $\nu = 1.5$, 10 and 100 cSt, respectively, and with a constant radius $R = 0.765$ mm. Depending on the forcing frequency, various modes are observed, that may be related to spherical harmonics Y_ℓ^m.

is observed. This high-viscosity behaviour has already been observed by Couder *et al* [1] for 500 cSt droplets bouncing on a 500 cSt bath. Those authors proposed to model the threshold curve by $\Gamma_{th} = 1 + \alpha f^2$, where α depends on the droplet size among other things. This equation is obtained by considering only the motion of the mass centre of the droplet and the squeezing of the air film between the droplet and the bath, without considering the droplet deformation. This model is valid for high viscosity regimes in contrast with the model developed in [7], where droplet deformations are taken into account in the lubrication force. According to this model, minima in Γ_{th} curves correspond to a resonance phenomenon of the droplet–air film system, as described in the introduction.

2. Resonant states

In 1879, Lord Rayleigh [8] described natural oscillations of an inviscid droplet. Since those oscillations are due to surface tension, natural frequencies scale as the capillary frequency $f_c = \sqrt{\sigma/M}$, where $M = (4\pi/3)\rho R^3$ is the droplet mass. More exactly, the dispersion relation prescribes the natural 'Rayleigh' frequency f_R related to an ℓ mode:

$$\left(\frac{f_R(\ell)}{f_c}\right)^2 = F(\ell) = \frac{1}{3\pi}\ell(\ell-1)(\ell+2). \tag{1}$$

The function $F(\ell)$ may vary (frequencies are shifted by a multiplicative factor), depending on the way the droplet is excited [10, 11]. For free oscillations, equation (1) is degenerated according to the m parameter. In figure 3, threshold data obtained for various droplet sizes collapse on a single curve by using the Rayleigh scaling. Moreover, at the bouncing threshold, the vertical force resulting from the droplet deformation exactly balances the gravity. One
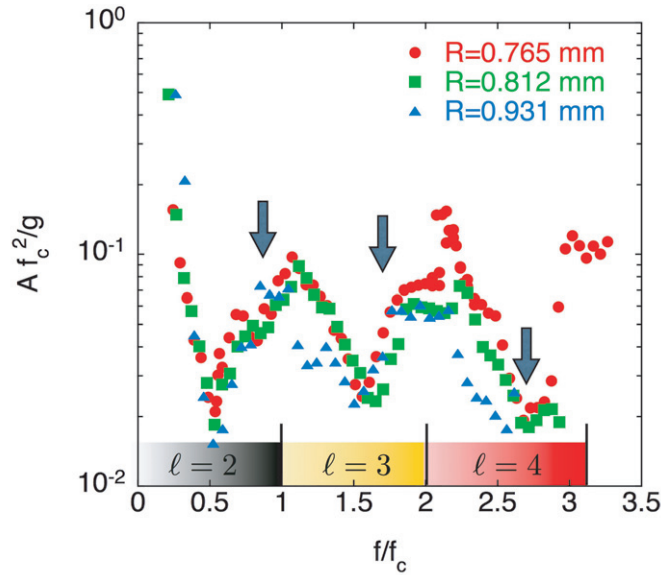
Figure 3. Dimensionless amplitude $A_{th} f_c^2/g$ as a function of the reduced forcing frequency f/f_c, where A_{th} is the amplitude of the bath vertical motion corresponding to the reduced acceleration Γ_{th} and $f_c = \sqrt{\sigma/M}$ the capillary frequency of a droplet of mass $M = (4\pi/3)\rho R^3$. Red bullets, green squares and blue triangles correspond to droplet radius $R = 0.765$, 0.812 and 0.931 mm, respectively. The droplet viscosity is $\nu = 1.5$ cSt. Modes $\ell = 2$, $\ell = 3$ and $\ell = 4$ are observed in the black, yellow and red ranges of frequency. Boundaries between those zones correspond to maxima in the threshold curve. Moreover, those particular frequencies may be obtained by multiplying the Rayleigh natural frequencies (equation 1) by a factor of 1.15. Arrows indicate the Rayleigh frequencies, which cannot be directly related to inflections in the threshold curve.

may define a characteristic length $L = g/f_c^2$ corresponding to the free fall distance during the capillary time $1/f_c$. As shown in figure 3, the threshold amplitude $A_{th} = \Gamma_{th} g/(4\pi^2 f^2)$ scales as the length L, whatever the droplet size is. The minimum value of $A_{th} f_c^2/g$ does not vary significantly with the mode index ℓ. In figure 3, the natural frequencies defined in equation (1) and represented by arrows do not correspond either to the minima in threshold, or to the maxima. However, these frequencies multiplied by 1.15 give the maxima positions, at $f/f_c = 1.05$, 2.05 and 3.15 for $\ell = 2$, 3 and 4, respectively. This numerical factor depends on the geometry of the excitation mode (bouncing in this case): e.g. another (smaller) factor is obtained when the droplet is stuck on a vibrated solid surface [10, 11].

As shown in [7], the bouncing ability of droplets is due to the cooperation of (i) the droplet deformation that stores potential energy, and (ii) the vertical force resulting from the squeezing of the intervening air layer between the droplet and the bath. The forced motion of the bath provides some energy to the droplet, a part of which helps the droplet to bounce (translational energy) while the other part increases internal motions inside the droplet, which are eventually dissipated by viscosity. The proportion of energy supplied to the translational/internal motion varies with the forcing frequency (i.e. when the energy is provided in the oscillation cycle). Minima (respectively maxima) in the threshold curve correspond to a maximum (respectively
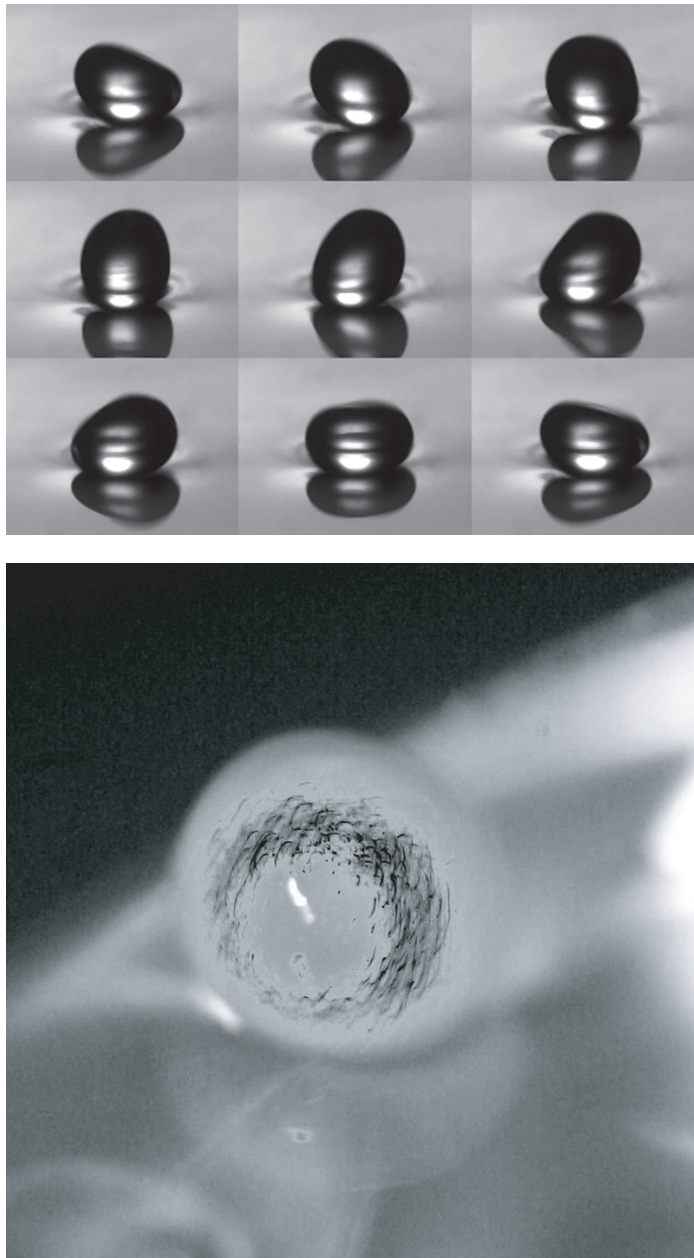
Figure 4. Rolling motion of the droplet (top): mode Y_2^1 of a bouncing droplet ($\nu = 1.5$ cSt, $R = 0.765$ mm) observed at $f = 115$ Hz and $\Gamma = 4.5 \geqslant \Gamma_{th}$. Frames are separated by 1 ms. The droplet is rolling towards the left. Bottom: the rolling motion is revealed by small bright particles spread inside the droplet. Movie 2 compares the droplet in the mode Y_2^1 and the mathematical function from MATLAB^{TM}. Movie 3 shows the motion of the particles inside the droplet when it rolls over the surface of the bath. Both movies are available from stacks.iop.org/NJP/10/113021/mmedia.

minimum) of the translational to internal energy ratio. Maxima may be related to the cut-off frequency recently observed and theoretically explained by Gilet *et al* [7]. This fact is confirmed experimentally, since the maxima in threshold correspond to the boundaries between modes. At
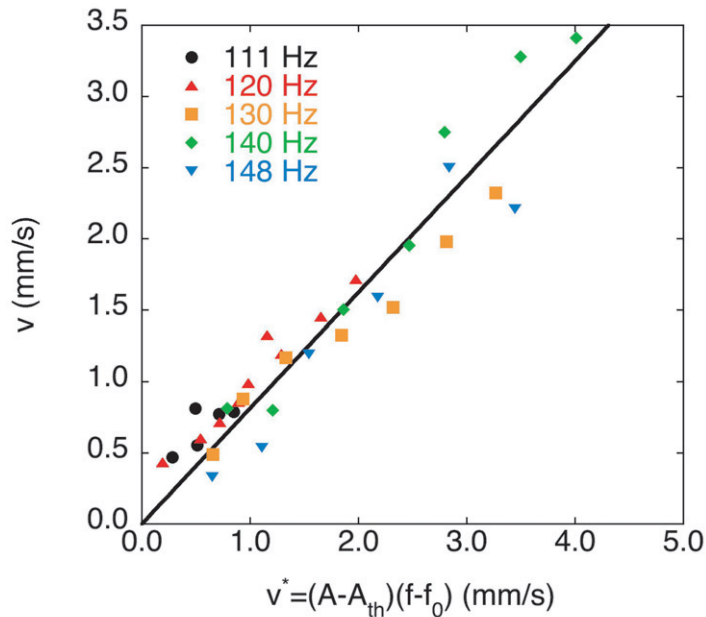
Figure 5. Initial speed of the roller for various frequencies (indicated in the legend) as a function of the characteristic speed v^*. The droplet radius is 0.765 mm while the viscosity is 1.5 cSt.

those points, corresponding to a forcing frequency of 1.15 f_R, most energy is spent in internal motions: the droplet resonates and absorbs energy.

3. Roller drop

The droplet behaviour has been observed in the vicinity of the first maximum in the threshold curve. At 115 Hz, the droplet moves along a linear trajectory analogously to the walkers observed in [2]. In this latter case, 50 cSt droplets were produced on a 50 cSt silicon oil bath. The mechanism for the walking motion is the interaction between the droplet and the bath surface wave generated by the bouncing. Such a mechanism does not hold in the case of a 1000 cSt silicon oil bath. Indeed, the generated waves are rapidly damped and cannot be responsible for the motion of the droplet. A movie of the moving droplet has been recorded using a high speed camera (figure 4). The images are separated by 1 ms. The deformation is not axi-symmetrical as in the resonant minima of the threshold acceleration curve. On the other hand, the mode is related to $\ell = 2$ and $m = 1$. This mode is characterized by two lines of nodes that are orthogonal. This results in the existence of two fixed points located on the equator of the droplet. As the lines of nodes do not follow the axi-symmetrical geometry, those lines move. More precisely, they turn giving the droplet a straight direction motion. This is a way to observe the Y_2^1 mode of deformation, which is generally degenerated with the Y_2^0 mode since equation (1) depends only on ℓ. The first maximum corresponds to the resonance of the Y_2^m mode. Due to the air film dynamics, a droplet in the Y_2^0 mode cannot bounce [7], while a droplet in the Y_2^1 mode bounces and rolls. As far as Rayleigh frequencies are concerned, we observe a break of degeneracy for the m parameter!

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Some tracers have been placed in the droplet. The reflection of the light on the particles allows the inner fluid motion to be followed, which clearly reveals that the droplet rolls over the bath surface. The initial speed v of the droplet has been measured with respect to the forcing parameters (amplitude and frequency). A scaling is found when considering that the phenomenon can occur only above a cut-off frequency $f_0 \approx 103$ Hz and the amplitude threshold $A_{\text{th}}(f) = \Gamma_{\text{th}}(f)g/(4\pi^2 f^2)$ as

$$v = 2\pi\alpha(A - A_{\text{th}}(f))(f - f_0), \tag{2}$$

where α is a constant. The initial speed is represented versus the characteristic speed $v^* = (A - A_{\text{th}}(f))(f - f_0)$ in figure 5. The proportionality between the initial speed of the roller and v^* is remarkable. Three characteristic frequencies are involved in the rolling process: (i) the excitation time 100 Hz, (ii) the rotation of the liquid inside the droplet about 10 Hz and (iii) the translational motion about 0.2 Hz (the droplet travels a distance corresponding to its circumference once every 5 s). The three processes are interrelated, i.e. the deformation induces the internal motion which induces the rolling and the translational motion.

4. Summary

The deformation of low viscosity bouncing droplets is emphasized on a high viscosity bath: the droplet oscillations are much less damped than the bath oscillations. Depending on the forcing frequency, droplets need a different amount of supplied energy to achieve a sustained periodic bouncing. When the forcing frequency corresponds to a multiple of the eigenfrequency of the bouncing droplet ($f \sim (\ell - 1)f_c$), the supplied energy is mainly lost into internal motions, so the threshold acceleration Γ_{th} is at a maximum. On the other hand, Γ_{th} is at a minimum when $f/f_c = \ell - 3/2$.

A new self-propelled mode has been discovered for forcing frequencies between the $\ell = 2$ and $\ell = 3$ modes (between roughly 100 and 150 Hz) for a sufficiently low droplet viscosity. A break of degeneracy is observed for the m parameter since the self-propelled mode corresponds to a non-axi-symmetrical mode Y_2^1. This mode is characterized by an internal rotation of the fluid inside the droplet. The droplet rolls over the vibrated bath! This droplet displacement technique is more adapted to large low-viscosity droplets; it is therefore of considerable interest for microfluidic applications: manipulating aqueous mixtures without touching them.

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