# Digital microfluidics on a wire

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In this letter, we discuss the behavior of droplets on fiber networks. An on/off transition is observed when a droplet comes around an intersection between several fibers: large droplets cross the junction while small droplets remain pinned. We show that fibers perform advantageously most operations of digital microfluidics, such as multiplexed biochemical microreactions: intersections are the basic component of fiber-based microfluidic devices. © *2009 American Institute of Physics.*
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Dew droplets are seen on spider webs in the early morning. Small ones remain pinned on the threads while larger ones slide down the grid, fuse with others, leave smaller droplets in their wake, and eventually fall from the web. Spontaneous motion, coalescence, and division are words commonly encountered in digital microfluidics, where droplets are used as tiny liquid entities manipulated through complex channel networks. The spontaneous formation of droplets on a fiber was reported by Plateau.[1] He observed that fluid coating is unstable on a fiber: the liquid minimizes its surface energy by forming a string of pearls instead of a film with constant thickness. This instability was later rationalized by Lord Rayleigh.[2] Since the pioneer work of Landau and Levich,[3] several studies have been devoted to fiber coating by liquids.[4] Much less work has been done on the behavior of a single droplet on a thread.[5–7] The shape and the motion ability of a droplet on a fiber depend on its wetting properties. An oil droplet (surface tension $\sigma \simeq 20$ mN/m) perfectly wets and slides down an inclined nylon fiber ($\sigma \simeq 46$ mN/m), while a pure water droplet ($\sigma \simeq 72$ mN/m) cannot usually do the same. It remains pinned on the fiber while its weight may be balanced by a bottom/up difference in contact angle.[8] Droplets that are small compared to the fiber diameter switch from a barrel to a clam-shell shape.[9]

We have laid silicon oil droplets (DC200) on vertical nylon fibers (fishing thread) by using a syringe. Fibers are cleaned and dried before each experiment. We have measured the velocity $V$ of droplets with various volumes $\Omega \sim 1$ $\mu$L and kinematic viscosity $\nu \in [10, 100]$ cS on fibers of radius $b_V \in [40, 125]$ $\mu$m. The gravity quickly accelerates the droplet until balanced by frictional forces due to the fluid viscosity. In our experiments, the droplet reaches its terminal velocity after $V/g \lesssim 4$ ms, i.e., $V^2/g \sim 0.2$ mm. As suggested in Ref. 7, the force balance writes

$$\Omega g \simeq c_F \nu b_V V. \qquad (1)$$

The proportionality factor $c_F$ is approximately equal to $23.2 r^{-1}(b_V \kappa)^{-1/2}$ where $\kappa^{-1} = \sqrt{\sigma/\rho g} \simeq 1.5$ mm is the capillary length and $r \sim 0.5$ is the aspect ratio (horizontal/vertical extension) of the droplet. This scaling law is in excellent agreement with experimental data, as seen in Fig. 1. The terminal velocity slowly decreases with time since the droplet loses some mass by coating the fiber (characteristic length about 50 mm in our experiments).

Things become much more interesting when the sliding droplet encounters an intersection with another fiber. A vertical and a horizontal fibers (radius $\in [40, 125]$ $\mu$m) are put together so they just touch each other; contact is ensured by the tension applied to the fibers. Droplets slide down the vertical fiber to reach the intersection. Small droplets remain pinned, while large ones cross the junction, leaving a tiny fraction on the node (Fig. 2). Small droplets are trapped due to the capillary attraction between the droplet and the horizontal fiber. On the other hand, gravity makes the droplet pass through. The ratio between those two forces is given by the Bond number,

$$\text{Bo} = \frac{mg}{4\pi\sigma b_H} = \frac{\kappa^2 \Omega}{4\pi b_H}, \qquad (2)$$

where $m$ is the droplet mass and $b_H$ is the radius of the horizontal fiber. Droplets of various sizes ($\Omega \sim 1$ $\mu$L) and viscosity ($\nu \in [1.5, 100]$ cS) have been laid on the vertical fiber. Both the initial volume of the droplet and the volume of liquid that has crossed the node are measured. The related Bond numbers (initial $\text{Bo}_i$ and final $\text{Bo}_f$) are plotted in Fig. 3(a) in the case of $b_H = 125$ $\mu$m and $b_V = 40$ $\mu$m. A sharp transition occurs between blocking ($\text{Bo}_f = 0$) and crossing ($0 < \text{Bo}_f \lesssim \text{Bo}_i$) regimes. The critical Bond value for transition is $\text{Bo}^* = 1.12$ at high viscosity ($\geq 20$ cS); it decreases
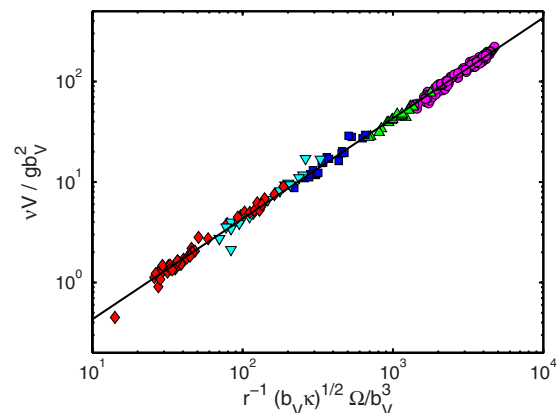


FIG. 1. (Color online) Terminal velocity $V$ of a droplet sliding down a vertical fiber as a function of its volume $\Omega$. Symbols represent different fiber radii, whatever the viscosity between 10 and 100 cS: (●) $b_V = 40$ $\mu$m, (▲) $b_V = 50$ $\mu$m, (■) $b_V = 70$ $\mu$m, (▼) $b_V = 100$ $\mu$m, and (♦) $b_V = 125$ $\mu$m. The solid line is the scaling law (1). Error bars typically correspond to the symbol size.

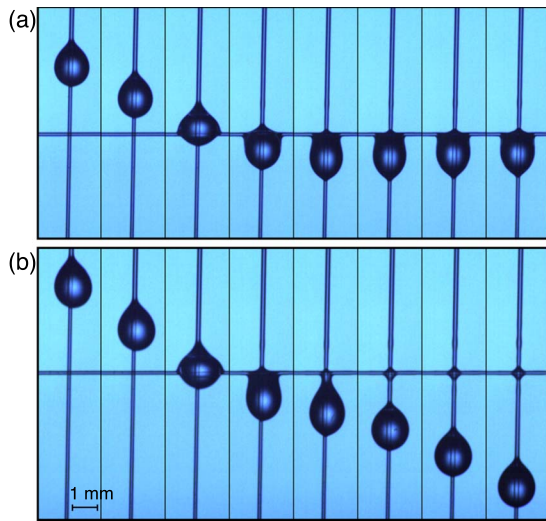a)Electronic mail: tristan.gilet@ulg.ac.be.

FIG. 2. (Color online) A 5 cS oil droplet interacts with a junction between two nylon fibers (radius 70 $\mu$m). Snapshots are taken every 10 ms. The intersection behaves as a fluidic diode: (a) a small droplet is pinned on the junction, while (b) a large droplet crosses it. In that latter case, a tiny amount of liquid is left at the intersection.
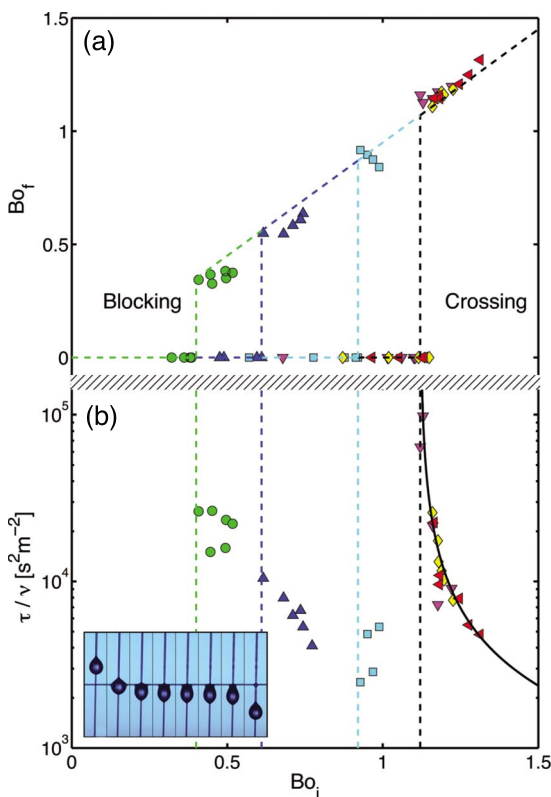


FIG. 3. (Color online) (a) Relationship between the volume of liquid allowed to cross the node and the initial droplet volume, expressed in terms of Bond numbers $Bo_f(Bo_i)$, where $Bo = \kappa^2 \Omega / 4 \pi b_H$. (Dashed lines) Equations $Bo_f = 0$ when $Bo_i < Bo^*$ (blocking) and $Bo_f = Bo_i - 0.05$ when $Bo_i > Bo^*$ (crossing). (b) Time $\tau$ needed by the droplet to cross the intersection, as a function of $Bo_i$. (Solid line) Empirical law $\tau / \nu = 900 / (Bo_i - Bo^*)$. [(a) and (b)] Experimental data for $b_H = 125$ $\mu$m and $b_V = 40$ $\mu$m: (●) $\nu = 1.5$ cS, (▲) $\nu = 5$ cS, (■) $\nu = 10$ cS, (▼) $\nu = 20$ cS, (◆) $\nu = 50$ cS, and (◀) $\nu = 100$ cS. Error bars typically correspond to the symbol size. (Inset) A delay of nearly 0.2 s is observed when a 50 cS droplet with $Bo \rightarrow Bo^*$ crosses the intersection between two fibers of radius 70 $\mu$m. Snapshots are taken every 34 ms.
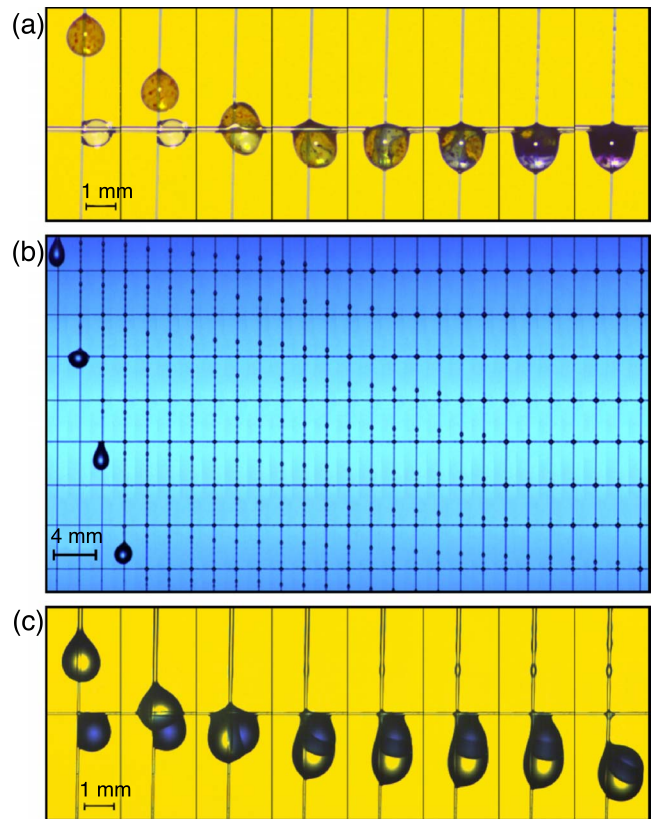


FIG. 4. (Color online) (a) A $H_2SO_4$ droplet (upcoming) fuses with a NaOH droplet on a node. The reaction is revealed by bromomethyl blue. The snapshot length is 0.1 s. (b) With nodes in series, a single large droplet is divided in many tiny droplets. The fluid lost by coating the vertical fiber is collected at junctions. Snapshots are taken every 48 ms. (c) A colored water droplet is placed on a fiber intersection. Then, a 50 cS oil droplet slides down the vertical fiber, wraps the water droplet, and the resulting liquid object leaves the junction. Snapshots are taken every 64 ms (enhanced online). [URL: http://dx.doi.org/10.1063/1.3157141.1]

when $\nu$ is decreased below 20 cS. In the high-viscosity regime (where $Bo^*$ is independent on $\nu$), the time spent to cross the intersection increases and diverges as $Bo \rightarrow Bo^*$, as often in physics when a potential barrier is just crossed. As seen in Fig. 3(b), the droplet may be significantly delayed; the delay time is proportional to the droplet viscosity.

This blocking/crossing sharp transition is particularly interesting to perform on droplets those elementary operations often encountered in microfluidic devices. In that sense, it is the analog of T-shaped junctions in channel-based microfluidics.[10] For example [Fig. 4(a)], small reactive droplets may meet together at a junction between fibers and perform a chemical reaction. By setting several nodes in series, a droplet can be divided into a large number of microdroplets, as easily as ever [Fig. 4(b)]. The large droplet crosses every intersection but leaves a tiny droplet at each of them. Moreover, the coating film on the vertical fiber transforms into pearls (Plateau instability) that are collected at each node. As an example, a single droplet ($\Omega \simeq 2$ $\mu$L) has been divided into 44 microdroplets on a network made of 40 $\mu$m fibers in about 1 s. Among others, it is also possible to encapsulate an aqueous droplet by an incoming oily droplet [Fig. 4(c)]. A number of applications require multiple microreactions to be performed in parallel, e.g., drug discovery, gene-expression analysis, and high-throughput assays.[10] With basic operations described in Fig. 4, multiplexing is

easily performed: the substance to be tested is placed on *N* nodes by using operation (b), then the (b)-device is rotated by 90° and markers are placed on the *N* vertical (previously horizontal) fibers. The biochemical reactions are made thanks to operation (a) and the result may (or not) flow down and be analyzed.

Fiber-based microfluidic devices present numerous advantages over existing technologies (pressure-driven droplet convection into microchannels, manipulation of droplets on a chip through electrowetting, etc.). The operations described here above are robust to the physico-chemical properties of the liquid of interest. They only require the liquid to wet the fiber, which is easily satisfied by using adequate fiber materials or by adding surfactant molecules. Contrary to the planar laboratories-on-a-chip, the geometry of fibers allows the design of fully three-dimensional networks with many fibers bringing multiple reactants to the same point. It also reduces the contact between droplets and solid parts; the loss of liquid through coating is minimized, especially since the coating pearls are also collected. Therefore, a millimetric droplet can be divided into tens of microdroplets on a fiber network. Thanks to the sharpness of the blocking/crossing transition, the volume of a droplet that leaves a node is accurately controlled. Although high-rate multiplexing may be performed with channel-based microfluidics[11], it often requires synchronization of the droplets conveyed into various channels, which is achieved through high-tech micropumps. Here, there is no need for any external synchronization: droplets wait for each other on the nodes. Moreover, there is no risk of denaturing the biochemical content of the droplet due to prohibitive electric fields generated by electronic components. Finally, fiber networks are simple to use, reusable, zero-energy consuming, and practically costless.

In this letter, we have discussed the behavior of droplets on fiber networks. The motion of a droplet on a fiber is driven by a balance between gravity and viscous friction. The observed long-term deceleration is due to the loss of mass by coating the fiber. When encountering an intersection between two fibers, small droplets remain pinned while large droplets cross the junction. The blocking/crossing transition occurs for a threshold volume, which results from a balance between gravity and capillary forces. With its specific behavior, the intersection between two fibers is the basic component of fiber-based microfluidics. Simple networks can perform elementary operations on droplets, such as coalescence, division, multiplexing, and encapsulation. The fiber-based microfluidics is proved to be a promising alternative to existing technologies that may be of interest for a number of biochemical, medical, and food-industry applications, e.g., low-cost medical diagnostic, DNA analysis, cell cultivation, drug discovery, and high-throughput assays.

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